

# Hawking-Moss Tunneling with a Dirac-Born-Infeld Action

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The Hawking-Moss tunneling rate for a field described by the Dirac-Born-Infeld action is calculated using a stochastic approach. We find that the effect of the non-trivial kinetic term is to enhance the tunneling rate, which can be exponentially significant. This result should be compared to the DBI enhancement found in the Coleman-de Luccia case.

Quantum tunneling in gravity is a well studied subject. The Coleman-de Luccia instanton [1] plays an important role in cosmology, e.g., in gauge theory phase transitions, in inflation and recently, in the cosmic landscape. In IIB string theory, where the extra dimensions are compactified, the motion of D3 branes play a crucial role in brane inflation. As is well-known, the kinetic term of a D3 brane is given by the Dirac-Born-Infeld action. Recently, Brown, Sarangi, Shlaer and Weltman showed that Coleman-de Luccia tunneling with a DBI action can be significantly enhanced when compared to that with a canonical kinetic term [2]. Hawking-Moss tunneling arises in the same contexts as Coleman-de Luccia tunneling when the potential barrier is broad instead of narrow. In many cases of interest the potential is not known. A natural question to ask is what is the impact of a DBI action on the Hawking-Moss tunneling rate. Potential applications include both inflation and vacuum selection [3], [4], [5]. During the observable stage of inflation observations disfavor large field space velocities for the inflaton in models with a DBI action [3]. For this reason we focus on the case where the field space velocity is small throughout the tunneling event.

Hawking and Moss used a Euclidean approach in [6] to calculate the tunneling rate in de Sitter space from a metastable state A to over a wide barrier with a maximum at B to the true minimum of the potential C. Two constant instanton solutions  $\phi(\tau, \mathbf{x}) = \phi_A$ ,  $\phi(\tau, \mathbf{x}) = \phi_B$  satisfy the classical equations of motion. Claiming that the tunneling rate is the exponential of the difference in the Euclidean action of these two instanton solutions  $P_{A \rightarrow C} \sim \exp(-(S(\phi_A) - S(\phi_B)))$  Hawking and Moss found the tunneling rate to be

$$P_{A \rightarrow C} \sim \exp \left( -\frac{3M_P^4}{8} \left( \frac{1}{V_A} - \frac{1}{V_B} \right) \right). \quad (1)$$

Several problems have been pointed out with this approach in the appendices of [7],[4]. The instanton solution  $\phi(\tau) = \phi_B$  does not interpolate between the metastable state and the true vacuum. However since the Euclidean scale factor  $b(\tau) = H^{-1} \sin(2\pi H\tau)$  vanishes at  $\tau = 0$  and  $\tau = H^{-1}$  large modifications can be made to the instanton solution  $\phi(\tau, \mathbf{x}) = \phi_B$  near the endpoints without changing the action significantly although the modified solution will no longer solve the classical equations of motion. This problem is particularly severe when there are

several metastable vacua between the initial metastable state and the true vacuum. Naively we could pick any of the instanton solutions sitting at a local maximum between the initial metastable state and the true vacuum to calculate the tunneling rate but generically the result depends on this choice of a local maximum. Additionally the actions  $S(\phi_A)$  and  $S(\phi_B)$  are both infinite since they are constant in all of Euclidean spacetime. The actions can be made finite by only considering tunneling in a Hubble-sized patch, but we then lose information about the rest of the universe. For these reasons we will not use the Euclidean approach of [6].

We find that the tunneling rate is modified to be

$$P_{A \rightarrow C} \sim \exp \left( \frac{3M_P^4}{8} \left( \frac{1}{\gamma V(\phi)} - \frac{(\gamma+1)(\gamma-1)}{2(V(\phi))^2} T(\phi) \right) \right) \Big|_{\phi_A}^{\phi_B} \quad (2)$$

when the inflaton  $\phi$  is described by the DBI action. Here  $H$  is the Hubble parameter given by  $H^2(\phi) = \frac{8\pi}{3M_P^2} \rho$ ,  $T$  is the warped brane tension, and

$$\gamma = \frac{1}{\sqrt{1 - \frac{\dot{\phi}^2}{T(\phi)}}} = \frac{1}{c_s} \geq 1 \quad (3)$$

where  $c_s$  is the sound speed. For small brane tensions  $T(\phi)/V(\phi) \ll 1$  the tunneling rate in Eq.(2) is enhanced over the rate in Eq.(1). In the limit that  $\gamma = 1$  Eq. (2) reduces to Eq.(1). Our results are valid in the limit that the curvature of the potential is small compared to the Hubble scale, that the energy density is dominated by the potential for the entire region in field space through which the tunneling occurs, and that the inflaton has a small field space velocity so that  $\gamma \gtrsim 1$ .

Instead of the Euclidean approach we will use the more clearly physically motivated stochastic approach of [8], [9], [10], [11]. The basic physical idea is that the quantum fluctuations of the short wavelength components of the inflaton field act as a random force on the long wavelength parts. If the inflaton is trapped in a metastable minimum, these fluctuations can drive the field up to a nearby maximum. From the maximum the inflaton has a probability of order unity to roll classically to a different minimum.

We consider a D3 brane probe moving in a type IIB

background described by the DBI action

$$S = - \int d^4x a^3(t) (T \sqrt{1 - \frac{\partial \phi^2}{T}} - T + V) \quad (4)$$

where  $T$  is the warped D3 brane tension,  $\phi$  is the canonically normalized inflaton, and  $V$  is the potential. In this theory  $\phi$  has a speed limit because of the non-minimal form of the kinetic term in Eq.(4).

We follow closely the treatment of [8]. The essential physical idea is that if we divide the field  $\phi(\mathbf{x})$  into a long-wavelength part  $\Phi$  and a short-wavelength part, the short-wavelength part acts as a random force on the long-wavelength part.

The average value of the field  $\phi$  over the coordinate volume  $b^3$  is given by

$$\phi_b = \frac{1}{(2\pi)^{3/2}} \frac{1}{b^3} \int d^3x e^{-|\mathbf{x}|^2/2b^2} \phi(\mathbf{x}). \quad (5)$$

Assuming the metric is flat, we can use the momentum space expansion

$$\phi(\mathbf{x}) = \int \frac{d^3k}{(2\pi)^{3/2}} \{a_{\mathbf{k}} \phi_{\mathbf{k}} e^{i\mathbf{k} \cdot \mathbf{x}} + a_{\mathbf{k}}^\dagger \phi_{\mathbf{k}}^* e^{-i\mathbf{k} \cdot \mathbf{x}}\} \quad (6)$$

to obtain

$$\phi_b = \int \frac{d^3k}{(2\pi)^{3/2}} e^{-k^2 b^2/2} \{a_{\mathbf{k}} \phi_{\mathbf{k}} + a_{\mathbf{k}}^\dagger \phi_{\mathbf{k}}^*\}. \quad (7)$$

Here  $a_{\mathbf{k}}$  and  $a_{\mathbf{k}}^\dagger$  are the ordinary creation and annihilation operators satisfying  $[a_{\mathbf{k}}, a_{\mathbf{q}}^\dagger] = \delta(\mathbf{k} - \mathbf{q})$ . For calculational simplicity we replace Eq. (7) with

$$\phi_b = \int \frac{d^3k}{(2\pi)^{3/2}} \Theta(-k + b^{-1}) \{a_{\mathbf{k}} \phi_{\mathbf{k}} + a_{\mathbf{k}}^\dagger \phi_{\mathbf{k}}^*\}. \quad (8)$$

We are interested in the macroscopic evolution of the field  $\phi$  on a de Sitter space background with metric  $ds^2 = a^2(t)(dx^2 + dy^2 + dz^2)$ . A string theory construction of such a background metric is given in [7]. The coordinate length associated with the physical length scale  $\gtrsim H^{-1}$  of interest is  $b = \frac{1}{\epsilon a} H^{-1}$  where  $\epsilon \ll 1$ . A more precise restriction on  $\epsilon$  will be obtained below. If we let  $\Phi$  denote the value of  $\phi$  averaged over this volume, then

$$\Phi = \int \frac{d^3k}{(2\pi)^{3/2}} \Theta(-k + \epsilon a H) \{a_{\mathbf{k}} \phi_{\mathbf{k}} + a_{\mathbf{k}}^\dagger \phi_{\mathbf{k}}^*\} \quad (9)$$

and the rate of change of  $\Phi$  is

$$\begin{aligned} \dot{\Phi} &= \int \frac{d^3k}{(2\pi)^{3/2}} \Theta(-k + \epsilon a H) (a_{\mathbf{k}} \dot{\phi}_{\mathbf{k}} + a_{\mathbf{k}}^\dagger \dot{\phi}_{\mathbf{k}}^*) \\ &+ \epsilon a H^2 \int \frac{d^3k}{(2\pi)^{3/2}} \delta(-k + \epsilon a H) (a_{\mathbf{k}} \phi_{\mathbf{k}} + a_{\mathbf{k}}^\dagger \phi_{\mathbf{k}}^*) \\ &\equiv \int \frac{d^3k}{(2\pi)^{3/2}} \Theta(-k + \epsilon a H) \{a_{\mathbf{k}} \dot{\phi}_{\mathbf{k}} + a_{\mathbf{k}}^\dagger \dot{\phi}_{\mathbf{k}}^*\} + g(t). \end{aligned} \quad (10)$$

We will see that  $g(t)$  plays the role of a random force due to the short-wavelength parts of  $\phi$ .

To find the equation of motion of  $\phi$  we use the results of [13] to calculate the energy density. Using  $p = T - T \sqrt{1 - \frac{\dot{\phi}^2}{T}} - V$  and  $\rho = 2X \partial_X p - p$  where  $X = \frac{1}{2}(\nabla \phi)^2$  we obtain

$$\rho = 2\gamma X + T/\gamma - T + V. \quad (11)$$

From Eq.(11) and the Friedman equations we find that the equation of motion of  $\phi$  on the background is

$$\ddot{\phi} + \frac{3H}{\gamma^2} \dot{\phi} + (2 - \frac{3}{\gamma^2}) \frac{\nabla^2}{a^2} \phi + \frac{1}{\gamma^3} V' - \frac{(\gamma+1)(\gamma-1)}{2\gamma^2} T' = 0 \quad (12)$$

where  $'$  denotes a partial derivative with respect to  $\phi$ .

If  $\phi = \phi_0$  is a metastable state then we can write  $V(\phi) = V_0 + \frac{m^2}{2}(\phi - \phi_0)^2 - \frac{\lambda}{4}(\phi - \phi_0)^4$  in a neighborhood of  $\phi = \phi_0$ . Assuming  $T(\phi)$  is analytic in a neighborhood of  $\phi = \phi_0$  we write  $T(\phi) = T_3(\alpha_0 + \frac{\alpha_2}{2}(\phi - \phi_0)^2 + \dots)$ . Using these Taylor expansions we see that the mode  $\phi_k$  approximately satisfies the equation

$$\begin{aligned} \ddot{\phi}_k + \frac{3H}{\gamma^2} \dot{\phi}_k + (\frac{3}{\gamma^2} - 2) \frac{k^2}{a^2} \phi_k + \frac{1}{\gamma^3} (m^2 \phi_k - \lambda \langle \phi^2 \rangle \phi_k) \\ - \frac{(\gamma+1)(\gamma-1)}{2\gamma^2} T_3(\alpha_2 \phi_k + \dots) = 0 \end{aligned} \quad (13)$$

If the short-wavelength modes of  $\phi$  are to act as a stochastic force on the spatially averaged  $\Phi$ , then these modes should obey the equation of a free field. For modes with  $k \gg \epsilon a H$  only the first three terms are important if three conditions are satisfied:

- 1)  $m^2/\gamma^3 \ll (3/\gamma^2 - 2)k^2/a^2$
- 2)  $\lambda \langle \phi^2 \rangle / \gamma^3 \ll (3/\gamma^2 - 2)k^2/a^2$
- 3)  $\frac{(\gamma+1)(\gamma-1)}{2\gamma^2} T_3(\alpha_2 + \dots) \ll (3/\gamma^2 - 2)k^2/a^2$ .

The first two conditions restrict the shape of the potential for which our approximation is valid. These conditions are weaker than the conditions for the validity of the Hawking-Moss tunneling rate (1) obtained by [8]. Our conditions reduce to the conditions of [8] when  $\gamma = 1$ . Condition 3 is always valid for small field space velocities (sufficiently close to  $\gamma = 1$ ) and we know that  $\gamma \gtrsim 1$  in a metastable state. Condition 2 is always valid in the neighborhood of a metastable state because the effective square of the mass  $M^2 = m^2 - \lambda \langle \phi^2 \rangle$  is always positive. This positivity combined with condition 1 implies

$$\lambda \langle \phi^2 \rangle < m^2 \ll (3\gamma - 2\gamma^3)k^2/a^2 \quad (14)$$

By [13] the expectation value of  $\phi^2$  is given by

$$\langle \phi^2 \rangle = \frac{3H^4}{8\gamma^2 \pi^2 m^2} \quad (15)$$

which gives us a bound on  $\lambda$

$$\lambda < \frac{8\pi^2}{3} \frac{m^4 \gamma^2}{H^4} \ll \frac{8\pi^2}{3} (3\gamma^3 - 2\gamma^5) \epsilon^4. \quad (16)$$

When our three conditions are satisfied the modes with  $k \gtrsim \epsilon a H$  satisfy the equation

$$\ddot{\phi}_k + \frac{3H}{\gamma^2} \dot{\phi}_k + \left(\frac{3}{\gamma^2} - 2\right) \frac{k^2}{a^2} \phi_k = 0 \quad (17)$$

where  $a = e^{Ht}$ . Using the results of [13]

$$\phi_k \approx -i \frac{H}{\gamma(2k^3)^{1/2}} \quad (18)$$

at the sound horizon. Substituting the expansion of  $\phi$  in momentum space into our equation of motion we find

$$\begin{aligned} & \int \frac{d^3 k}{(2\pi)^{3/2}} \\ & \{a_{\mathbf{k}}(\ddot{\phi}_{\mathbf{k}} + \frac{3H}{\gamma^2} \dot{\phi}_{\mathbf{k}} + (\frac{3}{\gamma^2} - 2)\phi_{\mathbf{k}})e^{i\mathbf{k}\cdot\mathbf{x}} + \text{H.C.}\} \\ & + \frac{1}{\gamma^3} V' - \frac{(\gamma+1)(\gamma-1)}{2\gamma^2} T' = 0 \end{aligned} \quad (19)$$

The integral in Eq.(19) is equal to

$$\begin{aligned} & \int \frac{d^3 k}{(2\pi)^{3/2}} \Theta(\epsilon a H - k) \{a_{\mathbf{k}}(\ddot{\phi}_{\mathbf{k}} + \frac{3H}{\gamma^2} \dot{\phi}_{\mathbf{k}} + \\ & (\frac{3}{\gamma^2} - 2)\phi_{\mathbf{k}})e^{i\mathbf{k}\cdot\mathbf{x}} + \text{H.C.}\} \end{aligned} \quad (20)$$

because

$$\begin{aligned} & \int \frac{d^3 k}{(2\pi)^{3/2}} \Theta(k - \epsilon a H) \{a_{\mathbf{k}}(\ddot{\phi}_{\mathbf{k}} + \frac{3H}{\gamma^2} \dot{\phi}_{\mathbf{k}} + \\ & (\frac{3}{\gamma^2} - 2)\phi_{\mathbf{k}})e^{i\mathbf{k}\cdot\mathbf{x}} + \text{H.C.}\} \end{aligned} \quad (21)$$

vanishes identically by Eq.(17). In Eq. (19) the first derivative term is the most important if  $\gamma \gtrsim 1$  which must be the case if the inflaton starts out in a metastable state. We assume  $\gamma \gtrsim 1$  for the entire region in field space through which tunneling occurs. Thus

$$\begin{aligned} & \int \frac{d^3 k}{(2\pi)^{3/2}} \Theta(\epsilon a H - k) \{a_{\mathbf{k}} \dot{\phi}_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}} + \text{H.C.}\} \\ & = -\frac{1}{3H} \left( \frac{1}{\gamma} V' - \frac{(\gamma+1)(\gamma-1)}{2} T' \right) \end{aligned} \quad (22)$$

Averaging this equation over the volume  $b^3$  we find that

$$\dot{\Phi} = -\frac{1}{3H} \left( \frac{1}{\gamma} \frac{\partial V}{\partial \Phi} - \frac{(\gamma+1)(\gamma-1)}{2} \frac{\partial T}{\partial \Phi} \right) + g(t) \quad (23)$$

where

$$g(t) = \epsilon a H^2 \int \frac{d^3 k}{(2\pi)^{3/2}} \delta(-k + \epsilon a H) \{a_{\mathbf{k}} \phi_{\mathbf{k}} + a_{\mathbf{k}}^\dagger \phi_{\mathbf{k}}^*\}. \quad (24)$$

Equation (23) is a Langevin equation with a random force  $g(t)$ . The correlation functions  $\langle g(t) \rangle$ ,  $\langle g(t_1)g(t_2) \rangle$ , etc. characterize the statistical properties of  $g(t)$ . We

compute these functions by averaging over the vacuum state  $|\rangle$  that satisfies  $a_{\mathbf{k}}|\rangle = 0$ . Clearly all of the odd correlation functions vanish. The two-point correlation function is given by

$$\begin{aligned} \langle g(t_1)g(t_2) \rangle &= \epsilon^2 H^4 a_1 a_2 \int \frac{d^3 k d^3 q}{(2\pi)^3} \delta(k - \epsilon a_1 H) \delta(q - \epsilon a_2 H) \\ & \langle a_{\mathbf{k}} a_{\mathbf{q}}^\dagger \rangle \phi_{\mathbf{k}} \phi_{\mathbf{q}}^* \\ &= \frac{\epsilon^2 H^6 a_1 a_2}{(2\pi)^2 \gamma^2} \int \frac{dk}{k} \delta(k - \epsilon a_1 H) \delta(k - \epsilon a_2 H) \\ &= \frac{\epsilon^2 H^6 a_1 a_2}{(2\pi)^2 \gamma^2} \frac{1}{\epsilon a_1 H} \delta(\epsilon a_1 H - \epsilon a_2 H) \\ &= \frac{H^3}{4\pi^2 \gamma^2} \delta(t_1 - t_2) \end{aligned} \quad (25)$$

It can be shown by induction that  $\langle g(t_1) \cdots g(t_n) \rangle = \sum \prod \langle g(t_i)g(t_j) \rangle$  for all even  $n$  where the sum is taken over all possible products of two-point functions. Therefore  $g(t)$  is a Gaussian variable and Eq.(23) leads to the standard Fokker-Planck equation

$$\frac{\partial \rho}{\partial t} = \frac{\partial}{\partial \Phi} \left( \frac{1}{3H} \left( \frac{1}{\gamma} \frac{\partial V}{\partial \Phi} - \frac{(\gamma+1)(\gamma-1)}{2} \frac{\partial T}{\partial \Phi} \right) \rho \right) + D \frac{\partial^2 \rho}{\partial \Phi^2} \quad (26)$$

with  $D = H^3/8\pi^2 \gamma^2$ .

There is a finite probability for a particle initially in a metastable state at  $\phi = \phi_A$  to stochastically climb up the potential to a nearby maximum at  $\phi = \phi_B$ . Once at the maximum the particle has a probability of order unity to classically roll down to an adjacent minimum  $\phi = \phi_C$ . By integrating the Fokker-Planck equation

$$\frac{\partial \rho}{\partial t} = \frac{\partial}{\partial \Phi} \left( \frac{1}{3H} \frac{\partial V}{\partial \Phi} \rho \right) + D \frac{\partial^2 \rho}{\partial \Phi^2} \quad (27)$$

in [12] it was found that the mean time during which a particle initially at  $\phi = \phi_A$  passes over the barrier at  $\phi = \phi_B$  of height  $\Delta V$  is given by

$$\Delta t \sim \exp \left( \int_{\phi_A}^{\phi_B} d\phi \frac{\partial}{\partial \phi} \frac{V(\phi)}{3H(\phi)D(\phi)} \right) \quad (28)$$

up to some subexponential prefactors. Using the same technique and (26) we see that

$$\Delta t \sim \exp \left( \frac{8\pi^2}{3} \left( \int_{\phi_A}^{\phi_B} d\phi \frac{\partial}{\partial \phi} \left( \frac{V}{\gamma H^4} - \frac{(\gamma+1)(\gamma-1)}{2H^4} T \right) \right) \right) \quad (29)$$

if the argument of the exponential is large. Equivalently the probability per unit volume is given by

$$\begin{aligned} P_{A \rightarrow C} &\sim \exp(-B_{HMDBI}) \\ &\sim \exp \left( \frac{3M_P^4}{8} \left( \frac{1}{\gamma V} - \frac{(\gamma+1)(\gamma-1)}{2V^2} T \right) \right) \Big|_{\phi_A}^{\phi_B} \end{aligned} \quad (30)$$

so the effect of the DBI action is to modify the tunneling rate from the result of [6]

$$\begin{aligned} P_{A \rightarrow C} &\sim \exp(-B_{HM}) \\ &\sim \exp \left( -\frac{3M_P^4}{8} \left( \frac{1}{V(\phi_A)} - \frac{1}{V(\phi_B)} \right) \right). \end{aligned} \quad (31)$$

For example in a warped type IIB compactification with fluxes [7] and D7-branes wrapped on 4-cycles, the potential for a probe D3-brane can have discrete minima in the angular directions of the compact space [16]. If at a fixed radial position at the bottom of the throat the D3-brane is in a false vacuum, it can tunnel in an angular direction. If we have  $T = (1.0 * 10^{-5} M_P)^4$ ,  $V(\phi_A) = (3.00 * 10^{-4} M_P)^4$ ,  $V(\phi_B) = (3.01 * 10^{-4} M_P)^4$ ,  $\gamma_A = 1.02$ , and  $\gamma_B = 1.01$ , then  $B_{HM}/B_{HMDBI} \sim 3.91$  so tunneling is exponentially faster than one would expect using (31). This result could have important applications for inflation when the inflaton is described by the DBI action, or for vacuum selection in the landscape. We leave these implications to future work.

After completing this work the author became aware of

[17] in which a result is derived that agrees with Eq.(2) in the limit that  $T/V \ll 1$ . See also [18] for a related discussion.

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